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TRANSFER FUNCTIONS FOR RIGID RECTANGULAR FOUNDATIONS

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SUMMARY

Closed-form expressions and comprehensive numerical solutions are presented for the transfer functions of surface-supported, rigid, rectangular foundations excited by horizontally polarized, incoherent shear waves for which the motions are parallel to one of the foundation sides. The free-field ground motion is specified stochastically in terms of a local power spectral density function and an orthotropic incoherence function which decays exponentially with the square of the excitation frequency and the separation distance. The response quantities examined include the lateral and torsional components of the foundation motion. Displayed graphically, the results elucidate the effects and relative importance of the numerous parameters involved. For vertically incident incoherent wave fields, the lateral transfer function of a rectangular foundation is related to that of a judiciously selected square foundation, and the interrelationship of the results is examined.

KEY WORDS: rectangular foundations; dynamics; transfer functions; incoherence

INTRODUCTION

A fundamental step in the analysis of the dynamic response of a foundation or of a foundation–structure system to spatially varying ground motions is the evaluation of the transfer functions of the foundation. Defined for harmonically excited massless foundations, these functions represent the ratios of the amplitudes of the components of the steady-state motion actually experienced by the foundation to the amplitude of the free-field ground motion at some reference or control point.

For surface-supported, rigid, circular foundations subjected to horizontal ground motions of a stochastically specified spatial variation, these functions have been evaluated by Prasad[1] and Veletsos and Prasad.[2] An approximate method of analysis, based on the averaging technique employed by Iguchi[3] and Scanlan[4] in their studies of wave passage effects for plane waves, was used. The objectives of the present study are to evaluate the corresponding functions for rigid, rectangular foundations, and to elucidate the effects and relative importance of the various parameters involved.

A unidirectional free-field ground motion directed parallel to one of the foundation sides is considered. The motion is characterized by a space-invariant power spectral density (PSD) function and an incoherence function of a particular form. The response quantities examined include the amplitudes of the lateral and torsional components of the resulting foundation motion; these amplitudes are evaluated over wide ranges of the parameters involved, and the more important trends are displayed graphically.

The problem examined here was studied previously by Luco and Wong,[5] who formulated the more nearly exact integral expressions for the transfer functions of the foundation, and presented numerical solutions for

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square foundations excited by vertically incident, incoherent shear waves. The advantages over the Luco–Wong formulation of the approximate method employed here are that: (a) it leads to relatively simple, closed-form expressions for the desired functions; and (b) it reduces the number of independent parameters that must be considered, thereby simplifying the interpretation of the resulting solutions. In view of the uncertainties involved in the characterization in practice of the free-field ground motion, the solutions presented here are believed to be sufficiently accurate for practical purposes. Some related aspects of the problem were also examined in References 6–16.

SYSTEM AND EXCITATION CONSIDERED

The system investigated is shown in Figure 1. It is a rigid, massless, rectangular foundation that is supported at the surface of an elastic half-space and is bonded to the supporting medium so that no sliding or uplifting may occur. The foundation is referred to a Cartesian system of co-ordinates, with its origin taken at the centre of the foundation, and its axes parallel to the foundation sides. The lengths of the foundation sides in the x- and y-directions are denoted by $2a$ and $2b$, respectively. The half-space is characterized by its shear modulus G, Poisson's ratio v, and mass density ρ. The shear wave velocity for the medium is then given by $v_s = \sqrt{G/\rho}$.

The free-field ground motion for all points of the foundation–soil interface is presumed to be directed along the x-axis, with the waves impinging on the foundation at an angle α_y with the vertical and propagating along the positive y-axis. The motion at any point is specified stochastically by a space-invariant, local PSD function $S_g(\omega)$, in which ω is the circular frequency of the harmonic component of the ground motion under consideration, and the spatial correlation of the amplitudes of the component motions at two arbitrary points defined by the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ is specified by the cross PSD function,

$$S(\mathbf{r}_1, \mathbf{r}_2, \omega) = \Gamma(\mathbf{r}_1, \mathbf{r}_2, \omega) \exp\left[-\mathrm{i}\omega\left(\frac{y_1 - y_2}{c_y}\right)\right] S_g(\omega) \tag{1}$$

Referred to as the incoherence function, the quantity Γ is a dimensionless, generally decreasing function of ω and of the distance between points; $\mathrm{i} = \sqrt{-1}$; y_1 and y_2 are the components (projections) of $\mathbf{r}_1$ and $\mathbf{r}_2$ in the direction of propagation of the waves; and c_y is the apparent horizontal velocity of propagation of the waves. The latter quantity is related to the velocity of shear wave propagation in the medium, v_s, by the expression

$$c_y = \frac{v_s}{\sin \alpha_y} \tag{2}$$

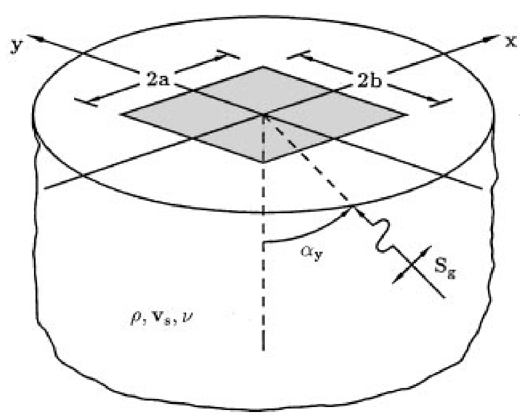

Figure 1. System considered

The product of S_g and the exponential term in equation (1) represents the component of ground motion variability associated with the wave passage effect, while the product ΓS_g represents the component due to the incoherence of the component waves. The peak value of Γ is unity and occurs at $\mathbf{r}_1 = \mathbf{r}_2$.

There is no general agreement at present on the form of the incoherence function that may be appropriate for earthquake ground motions, and several different expressions have been presented for this function.[5–13] For the solutions presented here, Γ is taken in the form proposed by Der *et al.*[13] as

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \omega) = \exp\left\{-\left(\frac{\omega}{v_s}\right)^2 [\gamma_x^2(x_1 - x_2)^2 + \gamma_y^2(y_1 - y_2)^2]\right\} \tag{3}$$

in which x_1 and x_2 are the x-co-ordinates of the points defined by the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$; y_1 and y_2 are the corresponding y-co-ordinates; and γ_x and γ_y are dimensionless factors with values typically in the range between 0 and 0·5. This form of incoherence will henceforth be referred to as *orthotropic*. For $\gamma_x = \gamma_y = \gamma$, equation (3) reduces to the *isotropic* form considered in References 1, 2, 5 and 6. Furthermore, for $\gamma_x = 0$ and $\gamma_y = \gamma$, it defines a one-dimensional incoherence in the y-direction; the motions of points with the same y-co-ordinate in this case are identical in both amplitude and phase.

EXPRESSIONS FOR TRANSFER FUNCTIONS

Let S_{ll} be the PSD function for the lateral or horizontal component of the foundation displacement, and S_{ss} be the corresponding function for the displacement component induced by the torsional component of the foundation motion along the edges parallel to the direction of the ground shaking. Denoted by $s(t)$, the latter component is defined by

$$s(t) = b\psi(t) \tag{4}$$

in which $\psi(t)$ represents the rotation of the foundation about a vertical centroidal axis at any time t, and b is the half-length of the foundation side normal to the direction of the free-field ground motion. With $s(t)$ defined in this manner, the effects of the lateral and torsional components of the foundation motion may be compared readily.

These functions were evaluated from the cross PSD function of the free-field ground motion by application of the averaging technique employed by Iguchi[3] and Scanlan.[4] This approach leads to

$$S_{ll} = \frac{1}{A^2}\int_A\int_A S(\mathbf{r}_1, \mathbf{r}_2, \omega)\,\mathrm{d}A_1\,\mathrm{d}A_2 \tag{5}$$

$$S_{ss} = b^2 S_{\psi\psi} = \frac{b^2}{I_\psi^2}\int_A\int_A y_1 y_2 S(\mathbf{r}_1, \mathbf{r}_2, \omega)\,\mathrm{d}A_1\,\mathrm{d}A_2 \tag{6}$$

The cross PSD function for the two components of the foundation motion, S_{ls}, is given similarly by

$$S_{ls} = bS_{l\psi} = \frac{b}{I_\psi A}\int_A\int_A y_2 S(\mathbf{r}_1, \mathbf{r}_2, \omega)\,\mathrm{d}A_1\,\mathrm{d}A_2 \tag{7}$$

in which $A = 4ab$ is the area of the foundation; $\mathrm{d}A_1$ and $\mathrm{d}A_2$ are elemental areas; and $I_\psi = \frac{1}{3}A(a^2 + b^2)$ is the polar second moment of area of the foundation about a vertical centroidal axis.

On introducing the dimensionless distances $\xi_1 = x_1/a$, $\xi_2 = x_2/a$, $\eta_1 = y_1/b$ and $\eta_2 = y_2/b$, and on making use of equation (1), equations (5)–(7) may be rewritten as

$$\frac{S_{ll}}{S_g} = \frac{1}{16}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \bar{\Gamma}\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2 \tag{8}$$

$$\frac{S_{ss}}{S_g} = \left(\frac{b^2 A}{I_\psi}\right)^2\left\{\frac{1}{16}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \eta_1\eta_2\bar{\Gamma}\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2\right\} \tag{9}$$

$$\frac{S_{ls}}{S_g} = \left(\frac{b^2 A}{I_\psi}\right)\left\{\frac{1}{16}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \eta_2\bar{\Gamma}\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2\right\} \tag{10}$$

in which

$$\bar{\Gamma} = \exp\{-[d_x^2(\xi_1 - \xi_2)^2 + d_y^2(\eta_1 - \eta_2)^2] - \mathrm{i}\, e_y(\eta_1 - \eta_2)\} \tag{11}$$

$$d_x = \gamma_x \frac{\omega a}{v_s} \tag{12}$$

$$d_y = \gamma_y \frac{\omega b}{v_s} \tag{13}$$

$$e_y = \frac{\omega b}{c_y} = \sin\alpha_y \frac{\omega b}{v_s} \tag{14}$$

Equations (8)–(10) can be integrated exactly in terms of the standard error function of complex argument to yield

$$\frac{S_{ll}}{S_g} = f_1(d_y, e_y)\, g_1(d_x) \tag{15}$$

$$\frac{S_{ss}}{S_g} = \frac{9}{[1 + (a/b)^2]^2} f_2(d_y, e_y)\, g_1(d_x) \tag{16}$$

$$\frac{S_{ls}}{S_g} = \frac{3}{1 + (a/b)^2} f_3(d_y, e_y)\, g_1(d_x) \tag{17}$$

in which the functions f_1, f_2, f_3 and g_1 are given by

$$f_1(d_y, e_y) = B_1(d_y, e_y) - B_3(d_y, e_y) - \frac{e_y}{4d_y^2} B_2(d_y, e_y) \tag{18}$$

$$f_2(d_y, e_y) = \frac{1}{3}\left\{B_1(d_y, e_y) - B_3(d_y, e_y) - \frac{1}{2d_y^2}[1 - B_3(d_y, e_y)] \right.$$
$$\left. - \frac{e_y(24d_y^4 - 6d_y^2 + e_y^2)}{32d_y^6} B_2(d_y, e_y) - \frac{e_y^2}{8d_y^4}[B_3(d_y, e_y) - 2B_4(d_y, e_y)]\right\} \tag{19}$$

$$f_3(d_y, e_y) = \mathrm{i}\left\{\frac{e_y}{4d_y^2}[B_1(d_y, e_y) - B_3(d_y, e_y)] + \frac{2d_y^2 - e_y^2}{16d_y^4} B_2(d_y, e_y)\right\} \tag{20}$$

$$g_1(d_x) = f_1(d_x, 0) = \frac{\sqrt{\pi}}{2d_x}\Phi(2d_x) - \frac{1 - \exp(-4d_x^2)}{4d_x^2} \tag{21}$$

and

$$B_1(d_y, e_y) = \frac{\sqrt{\pi}}{2d_y}\exp\left(-\frac{e_y^2}{4d_y^2}\right)\Re\left[\Phi\left(2d_y + \mathrm{i}\frac{e_y}{2d_y}\right)\right] \tag{22}$$

$$B_2(d_y, e_y) = \frac{\sqrt{\pi}}{2d_y}\exp\left(-\frac{e_y^2}{4d_y^2}\right)\Im\left[\Phi\left(2\mathrm{d}_y + \mathrm{i}\frac{e_y}{2d_y}\right) - \Phi\left(\mathrm{i}\frac{e_y}{2d_y}\right)\right] \tag{23}$$

$$B_3(d_y, e_y) = \frac{1 - \exp(-4d_y^2)\cos(2e_y)}{4d_y^2} \tag{24}$$

$$B_4(d_y, e_y) = \frac{\exp(-4d_y^2)\sin(2e_y)}{2e_y} \tag{25}$$

The function $\Phi(z)$ in equations (21)–(23) is the error function defined by

$$\Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-u^2)\,\mathrm{d}u \tag{26}$$

and the symbols $\Re[\cdot]$ and $\Im[\cdot]$ represent the real and imaginary parts of the bracketed quantities.

The quantities $\sqrt{S_{ll}/S_g}$ and $\sqrt{S_{ss}/S_g}$ represent the transfer functions for the lateral and torsional components of the foundation motion, whereas the modulus or amplitude of $S_{ls}/\sqrt{S_{ll}S_{ss}}$ is a measure of the degree of correlation between the two components of the motion. A numerical value of unity for the latter quantity indicates that the component motions are fully correlated, i.e with one component known, the other can be predicted, whereas a zero value indicates that the component motions are uncorrelated, i.e. their interrelationship is random.

Equation (15) reveals that the lateral transfer function, $\sqrt{S_{ll}/S_g}$, depends on the parameters d_x, d_y and e_y. When normalized with respect to the factor

$$C_o = \frac{3}{1 + (a/b)^2} \tag{27}$$

the transfer function for the torsional component of foundation motion also depends on the same three parameters (see equation (16)). By contrast, the amplitude of $S_{ls}/\sqrt{S_{ll}S_{ss}}$, which from equations (15)–(17) can be shown to be given by

$$\left|\frac{S_{ls}}{\sqrt{S_{ll}S_{ss}}}\right| = \frac{|f_3(d_y, e_y)|}{\sqrt{f_1(d_y, e_y)\, f_2(d_y, e_y)}} \tag{28}$$

is independent of d_x and a function only of d_y and e_y. The symbol $|\cdot|$ represents the amplitude of the enclosed quantity. Considering that the free-field ground motion is presumed to be directed along the x-axis, the motions in that direction for points with the same x-co-ordinate but different y-co-ordinates are clearly independent of γ_x. The same is also true of the lateral and torsional components of the resulting foundation motions.

Reduction to special cases

For vertically incident incoherent waves, for which $\alpha_y = 0$ and hence $e_y = 0$, equations (15)–(17) reduce to

$$\frac{S_{ll}}{S_g} = g_1(d_y)\, g_1(d_x) \tag{29}$$

$$\frac{S_{ss}}{S_g} = \frac{C_o^2}{3}\left\{ g_1(d_y) - \frac{1}{2d_y^2}\left[1 - B(d_y)\right]\right\} g_1(d_x) \tag{30}$$

$$S_{ls} = 0 \tag{31}$$

in which

$$B(d_y) = B_3(d_y, 0) = \frac{1 - \exp(-4d_y^2)}{4d_y^2} \tag{32}$$

The value of $|S_{ls}\sqrt{S_{ll}S_{ss}}|$ in this case is zero, and $\sqrt{S_{ll}/S_g}$ and the quantity $\sqrt{S_{ss}S_g}/C_o$ depend on d_x and d_y only.

For obliquely incident coherent waves, for which $\gamma_x = \gamma_y = d_x = d_y = 0$, equations (15)–(17) can be expressed solely in terms of e_y as

$$\frac{S_{ll}}{S_g} = \left(\frac{\sin e_y}{e_y}\right)^2 \tag{33}$$

$$\frac{S_{ss}}{S_g} = C_o^2 \left[\frac{1}{e_y}\left(\frac{\sin e_y}{e_y} - \cos e_y\right)\right]^2 \tag{34}$$

$$\frac{S_{ls}}{S_g} = \mathrm{i}C_o\left(\frac{\sin e_y}{e_y}\right)\left[\frac{1}{e_y}\left(\frac{\sin e_y}{e_y} - \cos e_y\right)\right] \tag{35}$$

Note that the value of $|S_{ls}/\sqrt{S_{ll}S_{ss}}|$ is unity in this case. Equations (33)–(35) have been presented previously by Luco and Sotiropoulos.[14]

RESULTS FOR VERTICALLY INCIDENT INCOHERENT WAVES

The transfer function $\sqrt{S_{ll}/S_g}$ and the quantity $\sqrt{S_{ss}/S_g}/C_o$ are plotted in Figure 2 as a function of d_y for fixed values of

$$\varepsilon = \frac{d_x}{d_y} = \frac{\gamma_x a}{\gamma_y b} \tag{36}$$

Referred to as the effective ratio of foundation side lengths, the factor ε may also be viewed as a measure of the relative importance of the ground incoherence in the x- and y-directions. Note that the solution for orthotropic incoherence (i.e. values of γ_x and γ_y that are different) is governed by the same number of dimensionless parameters as that for isotropic incoherence (i.e. values of $\gamma_x = \gamma_y = \gamma$). As a matter of fact, the results for the former case may be obtained from those for the latter by merely changing the ratio of the side lengths of the foundation, a/b, so that the values of ε in the two cases are the same.

For ease of comparison with the transfer functions for the lateral component of foundation motion displayed in the upper part of Figure 2, the corresponding functions for the torsional component are replotted in Figure 3 without the normalizing factor C_o. These particular data are for isotropic incoherence only.

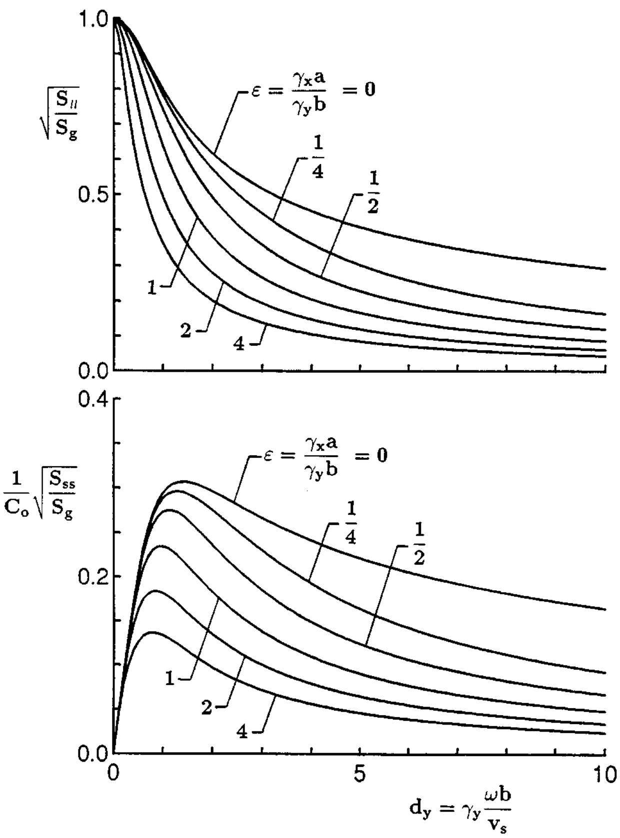

Figure 2. Normalized transfer functions for lateral and torsional components of foundation input motion for rectangular foundations subjected to vertically incident incoherent waves: plotted against frequency parameter d_y

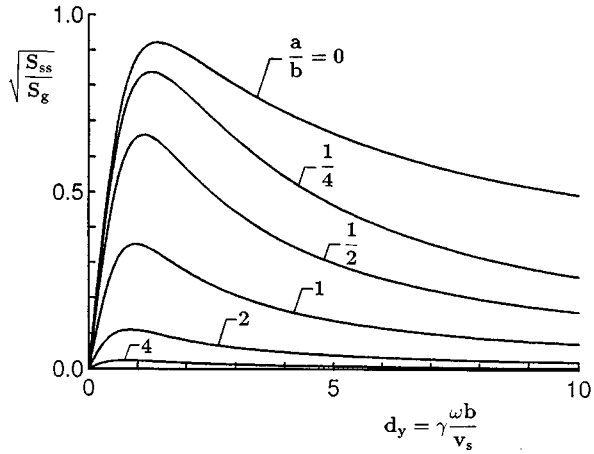

Figure 3. Transfer functions for torsional component of foundation input motion for rectangular foundations subjected to vertically incident incoherent waves: isotropic incoherence, $\gamma_x = \gamma_y = \gamma$

The following trends are worth noting in Figures 2 and 3:

1. For a fixed value of ε, the transfer function for the lateral component of foundation motion decreases monotonically with increasing d_y, whereas that for the torsional component increases from zero to a peak value and then decreases monotonically. These trends are similar to those of the corresponding functions for circular foundations presented in References 1 and 2.
2. Increasing the effective ratio of foundation sides, ε, decreases the values of both $\sqrt{S_{ll}/S_g}$ and $\sqrt{S_{ss}/S_g}$. These trends may be explained by referring to the results for isotropic incoherence, keeping in mind that the length b is effectively constant in these plots. Increasing a/b increases the length a over which the incoherence of the ground motion must be averaged, and this increase decreases the effective or weighted values of both the lateral and torsional components of the resulting foundation motion.
3. For isotropic incoherence ($\gamma_x = \gamma_y = \gamma$), the reductions with increasing a/b of the torsional component of foundation motion shown in Figure 3 are consistently greater than the corresponding reductions for the lateral component shown in the upper part of Figure 2. Two factors are responsible for this trend: (a) the torsional component of the foundation motion is more closely related to the second moment of area of the foundation about its vertical centroidal axis than is to its area, and (b) an increase in a increases the second moment of area of the foundation more rapidly than the area. The increase in the second moment of area is effectively represented by the factor C_o, defined by equation (27).
4. The curves for $\varepsilon = 0$ in Figures 2 and 3 are applicable to foundations of arbitrary ratios of sides, provided the ground motion incoherence is one-dimensional, i.e. $\gamma_x = 0$ and $\gamma_y \neq 0$. Because of the reduced interference between the components of the free-field ground motion in this case, the reduction with increasing d_y in the lateral component of foundation motion is smaller than for the two-dimensional incoherence, and the corresponding increase in the torsional component of motion is greater.

For the vertically incident incoherent wave fields examined in this section, the lateral and torsional components of the foundation motion are uncorrelated, i.e. $S_{ls} = 0$.

Alternative presentation of results

For the non-vertically incident waves considered in equations (15)–(17), the transfer functions were expressed in terms of d_x, d_y and e_y, whereas for the vertically incident waves examined in Figures 2 and 3, for which $e_y = 0$, they were expressed in terms of d_y and the effective side-length ratio, $\varepsilon = d_x/d_y$. In general, the

results can most effectively be displayed in terms of ε, the generalized frequency parameter,

$$\tilde{a}_o = \sqrt{d_x d_y + e_y^2} = \frac{\omega b}{v_s} \sqrt{\gamma_x \gamma_y \left(\frac{a}{b}\right) + \sin^2 \alpha_y} \tag{37}$$

and the generalized incoherence parameter,

$$\tilde{\gamma} = \frac{\sqrt{d_x d_y}}{e_y} = \frac{\sqrt{\gamma_x \gamma_y}}{\sin \alpha_y} \sqrt{\frac{a}{b}} \tag{38}$$

In addition to characterizing the frequency of the motion and the properties of the foundation and the supporting medium, the parameter $\tilde{a}_o$ is a measure of the overall spatial variation of the ground motion, whereas $\tilde{\gamma}$ is a measure of the relative importance of the components of that variation due to random incoherence and wage passage. These parameters are analogous to those used in the study of circular foundations reported in References 1 and 2. A value of $\tilde{\gamma} = 0$ refers to plane, coherent waves with arbitrary angle of incidence, α_y, whereas $\tilde{\gamma} = \infty$ refers to vertically incident, incoherent wave fields.

It is desirable at this stage to relate the behaviour of rectangular foundations of different proportions to that of an equivalent square foundation having the same area. On noting that the half-length of the side for the equivalent square foundation, b_e, is given by

$$b_e = \sqrt{ab} = b \sqrt{\frac{a}{b}} \tag{39}$$

equations (37) and (38) may be rewritten in the form

$$\tilde{a}_o = \frac{\omega b_e}{v_s} \sqrt{\gamma_x \gamma_y + (\sin^2 \alpha_y)(b/b_e)^2} \tag{40}$$

and

$$\tilde{\gamma} = \frac{\sqrt{\gamma_x \gamma_y}}{\sin \alpha_y} \frac{b_e}{b} \tag{41}$$

For the vertically incident incoherent waves examined in this section,

$$\tilde{a}_o = \sqrt{\gamma_x \gamma_y} \frac{\omega b_e}{v_s} \tag{42}$$

and $\tilde{\gamma} = \infty$. The parameters d_x and d_y are then related to $\tilde{a}_o$ and ε by

$$d_x = \tilde{a}_o \sqrt{\varepsilon} \tag{43}$$

and

$$d_y = \frac{\tilde{a}_o}{\sqrt{\varepsilon}} \tag{44}$$

The results presented in Figure 2 are replotted in Figure 4 as a function of $\tilde{a}_o$ for fixed values of ε. Note that when displayed in this format, the curves for the lateral transfer function almost merge into a single curve. Accordingly, this function may, to a reasonable degree of approximation, be considered to be independent of ε and expressible only in terms of the frequency parameter $\tilde{a}_o$, which, being a function of b_e, depends on the total area of the foundation. The high-frequency limit of the normalized version of the torsional transfer function is also nearly independent of ε, indicating that this limit too is a function of the total area of the foundation.

Also shown in Figure 4 by dashed lines is the lateral transfer function for rigid circular foundations and isotropic incoherence presented previously by Veletsos and Prasad.[2] The half-length of the side of the

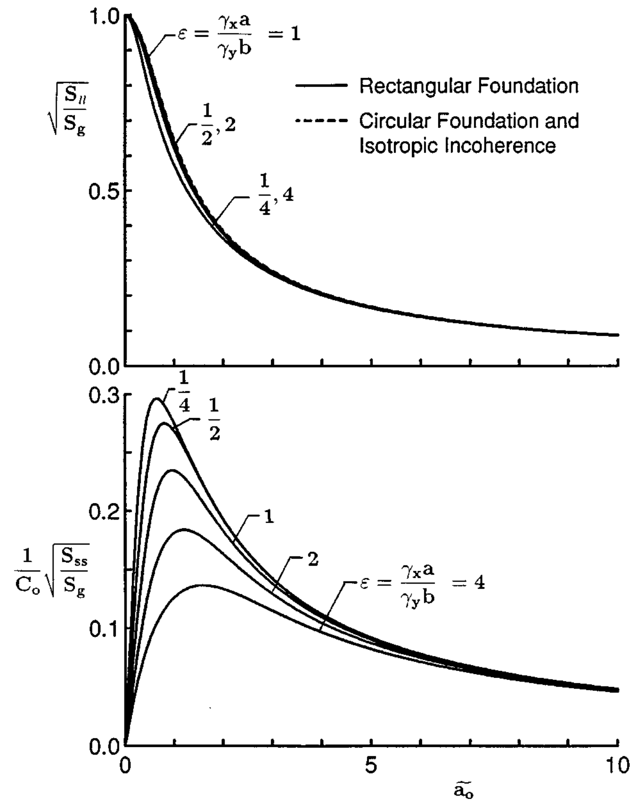

Figure 4. Normalized transfer functions for lateral and torsional components of foundation input motion for rectangular foundations subjected to vertically incident incoherent waves: plotted against frequency parameter $\tilde{a}_o$

equivalent square foundation in this case is

$$b_e = \frac{\sqrt{\pi}}{2} R \tag{45}$$

in which R is the radius of the circular foundation. The close agreement of the results for the circular and square foundations is a further confirmation of the dependence of the lateral transfer functions on the total area of the foundation.

An effort was also made to replot the results for the torsional transfer function in terms of parameters which might merge them into a single curve, but this objective could not be achieved. In particular, foundations with the same second moment of area about a vertical centroidal axis, I_ψ, did not generally yield comparable results. In retrospect, this result is not surprising, since the torsional component of the foundation motion depends not only on I_ψ, but also on the second moment of area of the foundation about a horizontal centroidal axis parallel to the direction of the free-field ground motion (see equation (9)).

Properties of lateral transfer functions

The following two properties may be established for the non-dimensionalized spectral density function of the lateral component of the foundation motion, S_{ll}/S_g:

1. On noting that equations (43) and (44) for d_x and d_y yield the same result if the value of ε is one equation is taken as its reciprocal in the other, it is concluded from equation (29) that

$$\left(\frac{S_{ll}}{S_g}\right)_\varepsilon = \left(\frac{S_{ll}}{S_g}\right)_{1/\varepsilon} \tag{46}$$

in which the subscripts identify the effective side-length ratios considered. It follows that, even for the orthotropic ground-motion incoherence, the transfer function for the lateral component of the foundation motion, $\sqrt{S_{ll}/S_{g}}$, is not altered by a 90° change in the orientation of the foundation.

2. On first squaring and then taking the square root of its right-hand members, equation (29) may, after regrouping of terms, be rewritten as

$$\frac{S_{ll}}{S_{g}} = \sqrt{\frac{S_{ll}}{S_{g}}(d_x, d_x)\frac{S_{ll}}{S_{g}}(d_y, d_y)} \tag{47}$$

The first term on the right-hand side of this equation represents the value of S_{ll}/S_{g} for a square foundation of side length $2a$ and isotropic incoherence characterized by a value of γ_x, whereas the second term represents the corresponding value for a square foundation of side length $2b$ and isotropic incoherence with γ_y. On identifying the two sets of conditions with the subscripts (a, γ_x) and (b, γ_y), equation (47) can be rewritten as

$$\frac{S_{ll}}{S_{g}} = \sqrt{\left(\frac{S_{ll}}{S_{g}}\right)_{a,\gamma_x}\left(\frac{S_{ll}}{S_{g}}\right)_{b,\gamma_y}} \tag{48}$$

In other words, S_{ll}/S_{g}, and hence the lateral transfer function $\sqrt{S_{ll}/S_{g}}$, for a rectangular foundation and *orthotropic* incoherence is equal to the geometric mean of the corresponding functions of square foundations with sides equal to each of the sides of the rectangular foundation and the indicated *isotropic* incoherences.

Note should finally be taken of the fact that as either a or γ_x tends to zero, $(S_{ll}/S_{g})_{a,\gamma_x}$ tends to unity, and hence

$$\frac{S_{ll}}{S_{g}} = \sqrt{\left(\frac{S_{ll}}{S_{g}}\right)_{b,\gamma_y}} \tag{49}$$

It follows that, both for very long, narrow foundations subjected to ground motions characterized by orthotropic incoherence, and for foundations of arbitrary proportions with one-dimensional incoherence defined by the parameter γ_y, the transfer function for lateral motion is equal to the square root of the corresponding function for a square foundation of side length $2b$ and isotropic incoherence with $\gamma = \gamma_y$.

RESULTS FOR OBLIQUELY INCIDENT INCOHERENT WAVES

Representative transfer functions for foundations subjected to obliquely incident incoherent waves are shown in Figure 5. Plotted for fixed values of $\tilde{\gamma}$ as a function of the generalized frequency parameter defined by equation (40), these curves are for systems with an effective ratio of side lengths $\varepsilon = 1$. As before, the transfer functions for the torsional component of the foundation motion are normalized by the factor C_{o}.

Whereas the curves for the larger values of $\tilde{\gamma}$ (i.e. when ground-motion incoherence dominates wave passage) vary smoothly in Figure 5, those for values of $\tilde{\gamma}$ equal to or close to zero (i.e. when wave passage dominates) are undulatory. These trends are similar to those presented in Reference 2 for circular foundations and isotropic incoherence. Incidentally, the curves for the limiting value of $\tilde{\gamma} = 0$ are independent of ε, and, therefore, also apply to a wider range of conditions than those considered for these plots.

The effect of the factor ε on the transfer functions of foundations subjected to obliquely incident waves is shown in Figure 6. These plots refer to systems with $\tilde{\gamma} = 1$. It is noteworthy that, unlike the corresponding plots for vertically incident incoherent waves ($\tilde{\gamma} = \infty$) presented in Figure 4, the curves for the lateral transfer functions in this figure do not merge into a single curve, neither do the high-frequency limits of the curves for the torsional transfer functions. The reason for these differences may be appreciated from equations (33) and (34), which define the wave passage effect for coherent, plane waves. Being functions of e_y, these expressions depend on the length b rather than the length b_{e} involved in the definition of $\tilde{a}_{o}$.

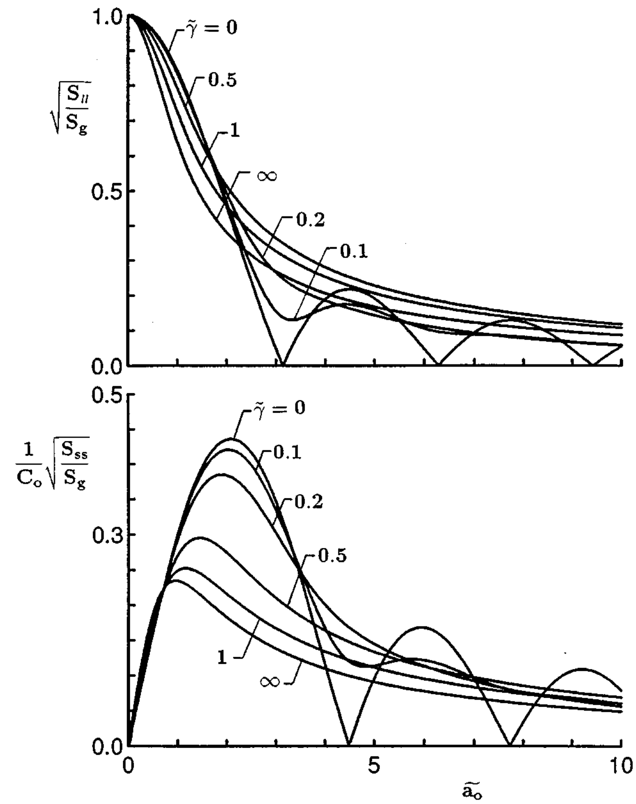

Figure 5. Normalized transfer functions for lateral and torsional components of foundation input motion for rectangular foundations with $\varepsilon = 1$

It may be recalled that the correlation amplitude for the lateral and torsional components of foundation motion, $|S_{ls}/\sqrt{S_{ll}S_{ss}}|$, depends on only two parameters, d_y and e_y. This amplitude is plotted in Figure 7 as a function of the modified frequency parameter

$$\hat{a}_o = \sqrt{d_y^2 + e_y^2} = \frac{\omega b}{v_s}\sqrt{\gamma_y^2 + \sin^2 \alpha_y} \tag{50}$$

and the modified incoherence parameter

$$\hat{\gamma} = \frac{d_y}{e_y} = \frac{\gamma_y}{\sin \alpha_y} \tag{51}$$

It is noteworthy that these results are independent of ε, and apply, therefore, to foundations of arbitrary ratio of sides and arbitrary values of the ground-motion incoherence parameter γ_x. The results in Figure 7 are similar to those presented in Reference 2 for circular foundations and isotropic incoherence.

EFFECT OF TWO-COMPONENT GROUND MOTION

The free-field ground motion in the discussion so far was presumed to be directed parallel to one of the foundation sides. For a two-dimensional excitation for which the components along the x- and y-axes may be considered to be independent of each other, the horizontal components of the foundation motion will also be independent of each other, the horizontal components of the foundation motion will also be independent, and the transfer functions for both the lateral and torsional responses can be obtained from the expressions already presented by superposing the effects of the component excitations. However, great care must be

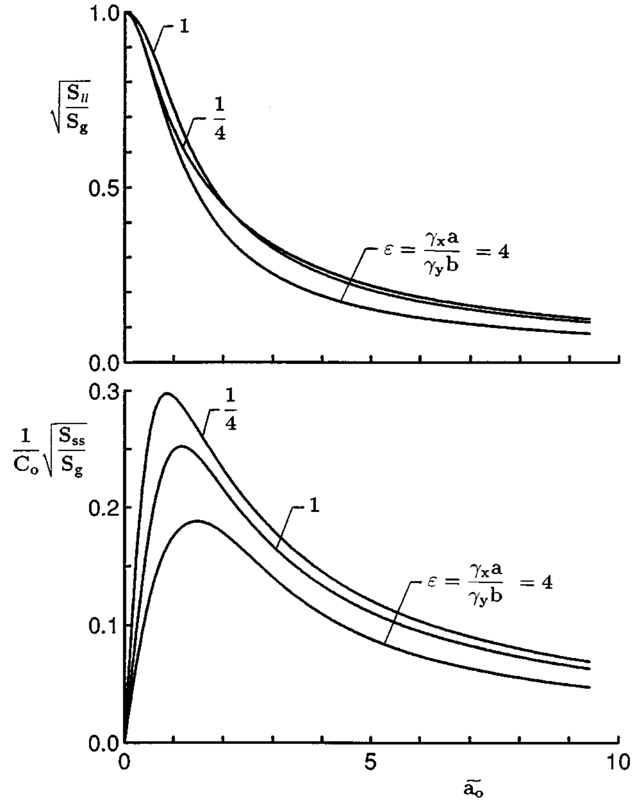

Figure 6. Normalized transfer functions for lateral and torsional components of foundation input motion for rectangular foundations subjected to obliquely incident incoherent waves with $\tilde{\gamma} = 1$

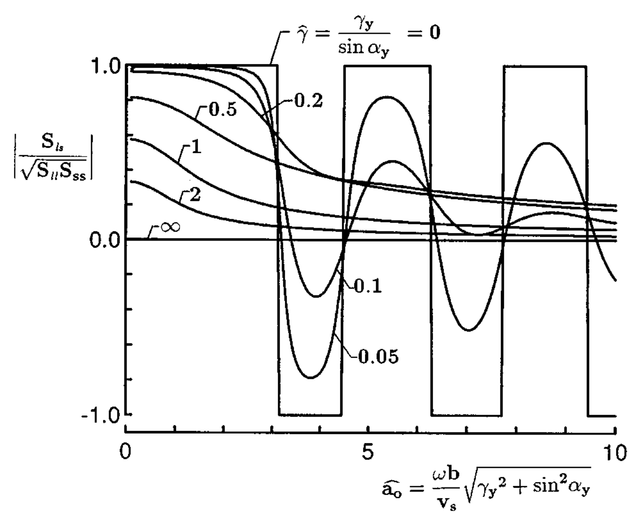

Figure 7. Normalized cross PSD functions for lateral and torsional components of foundation input motion for rectangular foundations subjected to obliquely incident incoherent waves

exercised in the application of these expressions properly to interpret the parameters involved. In this connection, it should be recalled that, for the one-dimensional excitation considered, the ground motion is directed parallel to the x-axis, the propagation of the waves is parallel to the y-axis, and the distances a and b are, respectively, parallel to and normal to the direction of the ground motion.

OTHER MEANINGS FOR RESULTS

Although defined specifically for the displacement histories of the foundation input motion, the spectral density ratios S_{ll}/S_{g}, S_{ss}/S_{g} and S_{ls}/S_{g} also define the ratios $S_{\dot{l}\dot{l}}/S_{\dot{\mathrm{g}}}$, $S_{\dot{s}\dot{s}}/S_{\dot{\mathrm{g}}}$, $S_{\dot{l}\dot{s}}/S_{\dot{\mathrm{g}}}$ and $S_{\ddot{l}\ddot{l}}/S_{\ddot{\mathrm{g}}}$, $S_{\ddot{s}\ddot{s}}/S_{\ddot{\mathrm{g}}}$, $S_{\ddot{l}\ddot{s}}/S_{\ddot{\mathrm{g}}}$ of the corresponding velocity and acceleration traces. It may be recalled that the PSD function for the first derivative of a stationary random process is given by the product of $(2\pi f)^2$ and the corresponding function of the original process.

CONCLUSION

Information and concepts have been presented which elucidate the effects and relative importance of the various factors that influence the transfer functions of surface-supported, rigid, rectangular foundations excited by horizontally polarized, incoherent shear waves with motions parallel to one of the foundation sides.

For vertically incident wave fields, the transfer function for the lateral component of foundation motion for a rectangular foundation of arbitrary proportions and orthotropic incoherence has been shown to be equal to the geometric mean of the corresponding functions for square foundations with sides equal to each of the sides of the rectangular foundation and appropriate isotropic incoherences. The lateral transfer function of a square foundation may, in turn, be approximated with good accuracy to that of an equivalent circular foundation. The transfer function for the torsional component of foundation motion is sensitive to the foundation side-length ratio, decreasing in value with increasing relative length of the foundation in the direction of the ground motion.

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